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[N185] Low Noise Design of Centrifugal Compressor Impeller

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ABSTRACT

The objective of this research is to suggest an efficient and robust optimization method for low-noise centrifugal compressor impeller design. Euler solver is used to simulate the flow-field of the centrifugal compressor and time-dependant pressure data are calculated to perform the near-field noise prediction by Ffowcs Williams-Hawkings formulation. Indirect Boundary Element Method is applied to consider the noise propagation effect. The optimization system consists of two categories; Response Surface Method to construct the response surface model based on D-optimal 3-level factorial design, and Genetic Algorithm to obtain the optimum solution of a defined objective function including penalty terms of constraints. The influence of design variables and their interactions on the aerodynamic noise is examined through the optimization process.

KEYWORDS: Centrifugal compressor impeller, Ffowcs Williams-Hawkings formulation, Indirect Boundary Element Method, Response Surface Method, Genetic Algorithm

INTRODUCTION

The experimental and analytical works have been conducted to understand the characteristics of the centrifugal compressor noise and to examine the influence of design parameters on the noise level [1-2]. From the results of them, it is observed that the Blade Passage Frequency (hereafter, BPF) noise in relation to the impeller rotation plays an important role [3]. Boundary Element Method (hereafter, BEM) solver coupled with Euler equations is developed to compute the BPF noise, and the optimization technique including Response Surface Method (hereafter, RSM) and Genetic Algorithm (hereafter, GA) is used to perform the low-noise design of the centrifugal compressor impeller.

The indirect variational BEM in the frequency domain is used to predict the inner and the outer noise propagation of the centrifugal compressor. The primary variables including the difference in the pressure and the difference in the normal gradient of the pressure contain information from the interior and the exterior acoustic space. The indirect formulation can be combined with a variational approach in deriving the primary system of equations [4].

RSM is the collection of statistical and mathematical techniques including design of experiment, regression analysis, and analysis of variance [5]. It has drawn much attention because of its efficiency and advantages; it smoothes out the high frequency noise to find the global optimum solution, and various objective functions and constraints can be attempted without additional calculations. The response surface model is constructed using flow analysis, and then GA method is adopted to find an optimum value [6].

NUMERICAL METHODS

Flow Analysis

The three-dimensional compressible Euler equations with the impeller moving grid and the patched grid are solved to analyze the flow unsteadiness due to circumferential inlet and outlet pressure distortions of the centrifugal compressor impeller. In the generalized coordinate system, the governing equation is

$$\frac{1}{J} \frac{\partial \hat{Q}}{\partial t} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} + \frac{\partial \hat{G}}{\partial \zeta} = 0 \quad (1)$$

The convective terms are discretized using Roe's flux difference splitting [7]. The primitive-variable extrapolation of the MUSCL approach is employed for higher-order spatial accuracy [8]. AF-ADI time marching method is adopted to calculate the unsteady flow-field of the rotating impeller [9].

Noise Prediction

Ffowcs Williams-Hawkings (hereafter, FW-H) equation of the point dipole assumption is used

to define the noise source of the centrifugal compressor impeller [10].

$$p(\vec{x}, t) = \frac{\cos\theta f}{4\pi} \left\{ \frac{i\omega}{rc} + \frac{1}{r^2} \right\} e^{i\omega(t-r/c)} \quad (2)$$

In Eq. (2), $p(\vec{x}, t)$ is the acoustic pressure, $\cos\theta$ is the directivity, f is the source strength, ω is the radiated frequency, r is the distance, and c is the speed of sound.

The boundary element method is based on expressing the acoustic pressure at a point within the acoustic medium as an integral over the boundary defining the acoustic domain, which is known as the Helmholtz integral equation [11].

$$Cp(\vec{r}_{dr}) = \int_S \left[G(\vec{r}, \vec{r}_{dr}) \frac{\partial p(\vec{r})}{\partial n} - \frac{\partial G(\vec{r}, \vec{r}_{dr})}{\partial n} p(\vec{r}) \right] dS \quad (3)$$

where \vec{r} is the point vector on the surface of the boundary element model, G denotes Green's function, \vec{r}_{dr} is the vector specifying the location of the data recovery point, and C is the integration constant resulting from the integration of Dirac's function originating from the fundamental solution to the governing differential wave equation.

In order to derive the integral equation for the indirect formulation, the standard approach used in indirect boundary element formulations is applied [12]. The integral equations for the two acoustic spaces are added together. Within the integral the terms which include Green's function are factored out and the opposite direction of the unit normal between the two equations is taken into account in generating the new primary variables. The equations for the primary variables and the acoustic pressure at a data recovery point are

$$\delta p = p_1 - p_2, \quad \delta dp = \left(\frac{\partial p}{\partial n} \right)_1 - \left(\frac{\partial p}{\partial n} \right)_2 \quad (4)$$

$$p(r_{dr}) = \int_S \left[\delta p(\vec{r}) \frac{\partial G(\vec{r}, \vec{r}_{dr})}{\partial n} - G(\vec{r}, \vec{r}_{dr}) \delta dp(\vec{r}) \right] dS \quad (5)$$

where δp is the difference in pressure between the two sides of the boundary and δdp is the difference in the normal gradient of the pressure.

Response Surface Method (RSM)

The response surface model is usually assumed as a second-order polynomial, which can be written for n_v design variables as follows.

$$y^{(p)} = c_0 + \sum_i c_i x_i + \sum_{1 \leq i \leq j \leq n_v} c_{ij} x_i x_j, \quad p = 1, \dots, n_s \quad (6)$$

The regression model of Eq. (6) is expressed in terms of an overdetermined matrix problem.

$$y = Xc$$

$$\begin{bmatrix} y^{(1)} \\ \vdots \\ y^{(n_s)} \end{bmatrix} = \begin{bmatrix} b_1(x^{(1)}) & \cdots & b_{n_{rc}}(x^{(1)}) \\ \vdots & \ddots & \vdots \\ b_1(x^{(n_s)}) & \cdots & b_{n_{rc}}(x^{(n_s)}) \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_{n_{rc}} \end{bmatrix} \quad (7)$$

In Eq. (7), $b_i(x)$ are basis functions and X is a $n_s \times n_{rc}$ matrix.

The regression coefficient matrix, c can be obtained to minimize the total statistical error using the least-squares fitting, which results in $c = (X^T X)^{-1} X^T y$. The number of regression coefficients, n_{rc} is $(n_v + 1)(n_v + 2) / 2$.

According to design of experiment, 3-level factorial design with the D-optimal condition is used as data-points selection technique. D-optimal criterion states that the data-points chosen are those that maximize the determinant, $|X^T X|$.

Analysis of variance and regression analysis are the statistical techniques to estimate regression coefficients in the quadratic polynomial model and yield the measure of uncertainty. One of important statistical parameters is the coefficient of determination, R^2 which measures how well the regression equation fits the data by flow analysis.

$$R^2 = \frac{SSR}{SSTO} = 1 - \frac{SSE}{SSTO} \quad (8)$$

In Eq. (8), SSTO is the total sum of squares, SSR is the regression sum of squares, and SSE is the error sum of squares. However, the **adjusted- R^2** statistic parameter is more important because a large value of R^2 does not necessarily imply that the regression model is good one.

$$R_{adj}^2 = 1 - \frac{SSE / (n_s - n_{rc})}{SSTO / (n_s - 1)} = 1 - \left(\frac{n_s - 1}{n_s - n_{rc}} \right) (1 - R^2) \quad (9)$$

For the accuracy of the response surface model, the significance testing of the individual regression coefficient is needed, which is provided by the **t-statistic** defined as

$$t = \frac{c_i}{\sqrt{\hat{\sigma}^2 (X^T X)^{-1}_{ii}}}, \quad i = 1, \dots, n_{rc} \quad (10)$$

In Eq. (10), $\hat{\sigma}^2$ is the estimation of variance. The coefficients with low value for the t-statistic are not accurately predicted.

RESULTS AND DISCUSSION

The centrifugal compressor model is used to demonstrate the noise prediction method and to perform the low-noise design of the impeller. Figure 1 shows the geometry of the centrifugal compressor impeller and the pressure contour by Euler analysis. Pressure fluctuations of the inlet and the outlet in the centrifugal compressor impeller resulting from the blade rotation are observed.

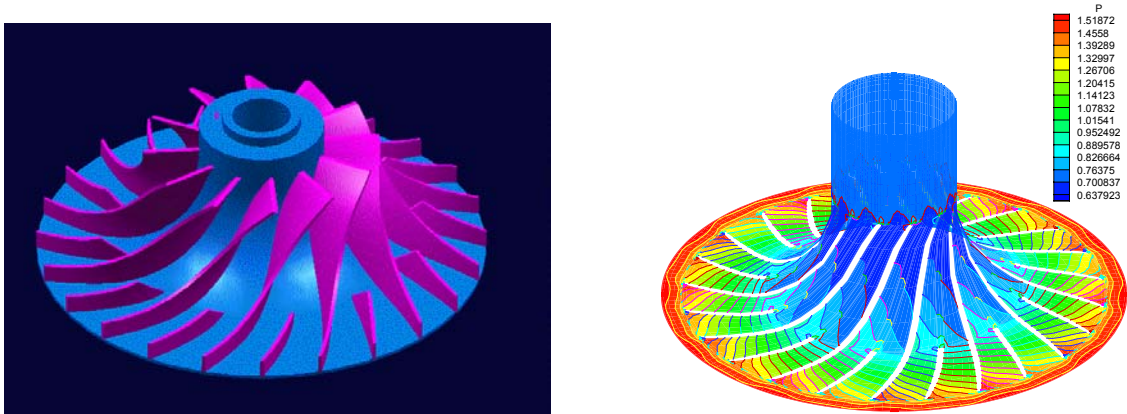


Figure 1: Centrifugal compressor impeller geometry and pressure contour computation

The estimation of regression coefficients and confidence level is accomplished by regression analysis and analysis of variance respectively to obtain the low-noise impeller geometry. The t-statistic computation gives the information about the relative importance of design variables. After building the response surface model, the objective function and constraint about the centrifugal compressor impeller performance and noise are applied to the GA method. The advancing impeller blade is obtained through the application of developed design technique.

CONCLUDING REMARKS

In the present study, the numerical prediction method of the centrifugal compressor noise and a new design procedure including RSM and GA are introduced. Euler solver is used to express the impeller flow-field and FW-H formulation is applied to estimate the noise source. Indirect variational BEM is employed to take the noise propagation into account. Although RSM has a limitation on the number of design variables due to computational cost, it is a viable optimization tool for low-noise centrifugal compressor impeller design.

REFERENCES

1. A. Stirnemann, H. Zogg, and F. Stocker, "Prediction of Turbo Compressor Noise," INTER-NOISE, Florida, USA, December 1999.
2. W. Jeon, and J. Lee, "A Study on the Noise Reduction Method of a Centrifugal Compressor," INTER-NOISE, Hague, Netherlands, August 2001.
3. M. Morgenroth, and D. S. Weaver, "Sound Generation by a Centrifugal Pump at Blade Passing Frequency," *Journal of Turbomachinery*, Vol. 120, 1998.
4. S. G. Mikhlin, *Variational Methods in Mathematical Physics*, MacMillan, New York, 1964.
5. R. H. Myers, and D. C. Montgomery, *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, John Wiley & Sons, New York, 1995.
6. D. E. Goldberg, *Genetic Algorithms in Search, Optimization & Machine Learning*, Addison-Wesley Publishing Company Inc., 1989.
7. P. L. Roe, "Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes," *Journal of Computational Physics*, Vol. 43, No. 3, 1981.
8. P. K. Sweby, "High Resolution TVD Schemes Using Flux Limiters," *Lectures in Applied Mathematics*, Vol. 22, 1985.
9. T. H. Pulliam, and D. S. Chaussee, "A Diagonal Form of an Implicit Approximate-Factorization Algorithm," *Journal of Computational Physics*, Vol. 39, No. 2, 1981.
10. A. P. Dowking, and J. E. Ffowcs Williams, *Sound and Sources of Sound*, John Wiley and Sons, 1983.
11. R. D. Cickowski, and C. A. Brebbia, *Boundary Element Methods in Acoustics*, Elsevier Applied Science, New York, 1991.
12. P. K. Banerjee, *The Boundary Element Methods in Engineering*, McGraw-Hill, New York, 1994.